Some Limitations on Parametric Amplifier Noise Performance*

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Summary—**Now that the precursory period following the solid-state parametric amplifier invention has given way to an era of determined effort to reduce to practice some of the early, optimistic predictions, practical limitations setting an upper bound to the performance of this ingenious communications device have become apparent. In this paper, two such limitations are discussed: First, for a given diode Q and junction geometry, there exists a noninfinite idler frequency, which determines the lowest radar noise temperature.[1] Second, because of its extreme noisiness the reverse-breakdown current limits the maximum capacitance swing at both extremes and consequently the minimum noise performance. It is suggested that in certain cases refrigeration may be a partial remedy to both limitations.**

I. Introduction

It IS commonly assumed that the minimum radar noise temperature of a parametric amplifier is obtained when the idler-to-signal-frequency ratio approaches infinity. While this notion is indeed a direct consequence of the energy conservation rules formulated by Manley and Rowe,[2] the presence of loss modifies this condition and a noninfinite ratio of idler to signal frequency results. Although the existence of this optimum ratio has already been pointed out,[3] it has not been adequately discussed in view of practical consequences. While in many cases this optimum ratio of idler to signal frequency is sufficiently large to demand more pump power than the diode can dissipate, there may be instances, especially at the lower frequencies, where some benefit can be derived from this knowledge. Since the detailed derivation of the expressions for noise temperature have been derived elsewhere,[3] they will be given here without proof as basis for further development.

II. Optimum Ratio of Idler to Signal Frequency

The radar noise temperature T_{n_1}, which in the most general case depends on circuit and diode losses at signal and idler frequencies as well as on the degree of coupling of an external idler termination, may be simplified by consideration of only the pertinent losses. These are the signal and idler diode conductances derived from the actual series equivalent circuit, under the assumption that Q_d is independent of frequency and $\gg 1$

$$G_{ds} = \frac{\omega_s \overline{C}}{Q_d}; \qquad G_{di} = \frac{\omega_s \overline{C}}{Q_d}\,\Omega^2. \tag{1}$$

Assuming a perfect circulator to separate input and output terminals in the signal port and high gain, one finds for the radar noise temperature

$$T_{n_1} \simeq \left[\frac{1}{\mu_{s_1}} + \frac{1}{\Omega}\left(1 + \frac{1}{\mu_{s_1}}\right)\right] T_d. \tag{2}$$

(All symbols are defined at the conclusion of the report.) The maximum μ_{s_1} for a given diode is determined by

$$\mu_{s_1} \leq \left(\frac{\Delta C}{2\overline{C}}\right)^2 \left(\frac{f_c}{f_s}\right)^2 \frac{1}{\Omega} - 1. \tag{3}$$

Substituting (3) into (2) with the additional definition,

$$K = \left(\frac{1}{2}\,\frac{\Delta C}{\overline{C}}\right) Q_d, \tag{4}$$

(hereafter referred to as diode parameter or figure-of-merit) shows the explicit dependence of T_{n_1} on the diode parameters and the idler-to-signal-frequency ratio Ω:

$$\frac{T_{n_1}}{T_d} \simeq \frac{1}{\dfrac{K^2}{\Omega} - 1} + \frac{1}{\Omega}\left\{1 + \frac{1}{\dfrac{K^2}{\Omega} - 1}\right\}. \tag{5}$$

The radar noise temperature exhibits a minimum value, where

$$\frac{\partial T_{n_1}}{\partial \Omega} = 0. \tag{6}$$

Carrying out the differentiation indicated in (6), one obtains the optimum idler-to-signal-frequency ratio

$$\Omega_{\text{opt}} = \sqrt{1 + K^2} - 1. \tag{7}$$

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[1] Radar noise temperature (figure) is suggested (and used henceforth) to indicate reception in the vicinity of one carrier frequency, even though one or more additional frequency ports may receive and generate thermal noise; the complementary method of reception using all available frequency ports will be christened radiometer noise temperature (figure). In the past, single-channel noise figure and double-channel noise figure, respectively, have often been used to describe the two methods of reception.

[2] J. M. Manley and H. E. Rowe, "Some general properties of nonlinear elements—pt. I. general energy relations," Proc. IRE, vol. 44, pp. 904–913; July, 1956.

[3] R. C. Knechtli and R. D. Weglein, "Low-noise parametric amplifier," Proc. IRE, vol. 48, pp. 1218–1226; July, 1960.

The minimum radar noise temperature that can be obtained from this diode (described by K) is

$$\left.\frac{T_{n_1}}{T_d}\right|_{\min} \geq \frac{\dfrac{K^2}{\sqrt{1+K^2}-1}+1}{K^2-\sqrt{1+K^2}+1}. \tag{5a}$$

In Figs. 1 and 2, the optimum Ω and the minimum T_{n_1} are plotted as a function of the diode parameter K; these curves are easily interpreted. Recalling the meaning of the diode parameter K from (4), one may now obtain an estimate of the lowest noise temperature that a given diode can produce under optimum conditions: These are optimum coupling conditions and no circuit residual losses. Fig. 3, showing T_{n_1} as a function of Ω, for several values of K, gives an estimate of the penalty one pays for operating at an Ω other than Ω_{opt}. Note that for large K the minimum T_{n_1} becomes less sensitive to Ω. The parameter K can be determined with relative ease; the maximum normalized capacitance swing can be computed for any type junction,[4] and the Q at the signal frequency can be measured by determining the cutoff frequency. Fig. 1 then shows the idler-to-signal-frequency ratio at which the minimum radar noise temperature is obtained, and Fig. 2 gives its value in terms of the diode body temperature T_d. It is interesting to observe that Fig. 1 shows solutions for the idler frequency lower than the signal frequency. This indicates that the diode parameter K is too small for such an application, if low noise is the objective.

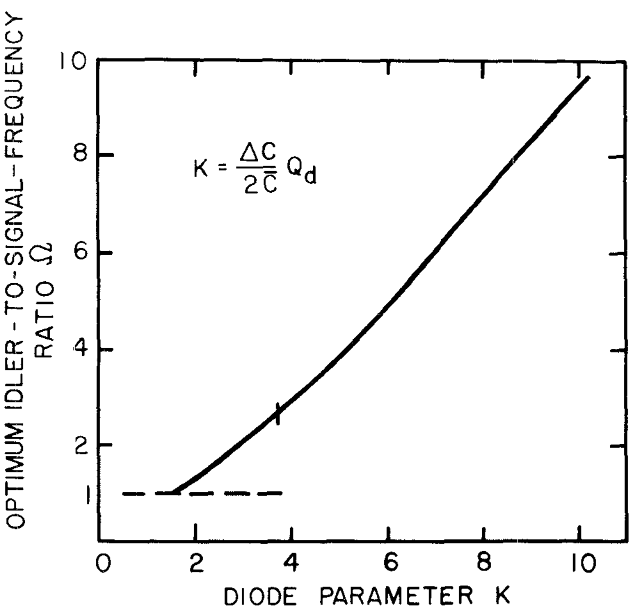

Fig. 1—Idler-to-signal-frequency ratio at which minimum noise temperature results.

If, on the other hand, the amplifier is operated as a radiometer and signal and idler frequencies are near each other and both are coupled to the source, a radiometer noise temperature can be defined. This is given by

$$T_{n_2} \simeq \frac{1}{\mu_{s_2}} T_d, \tag{8}$$

[4] S. Sensiper and R. D. Weglein, "Capacitance and charge coefficients for parametric diode devices," Proc. IRE, vol. 48, pp. 1482–1483; August, 1960.

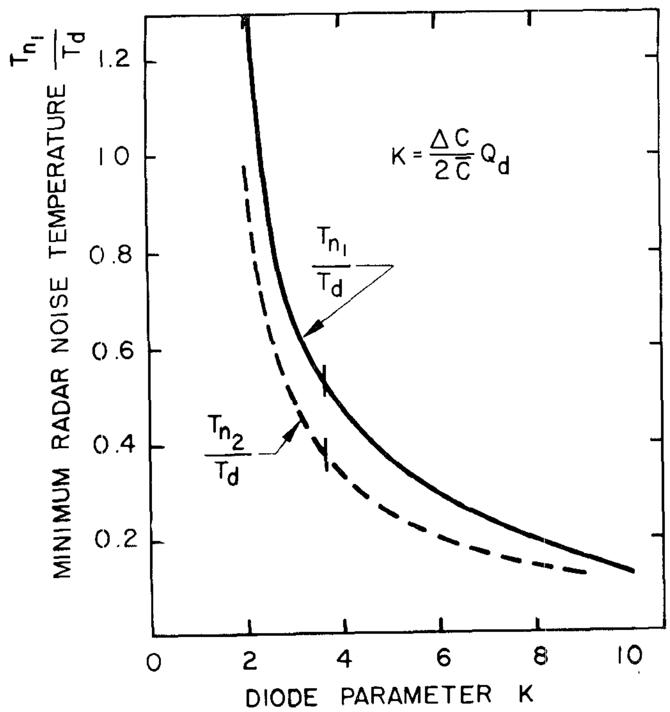

Fig. 2—Minimum noise temperature resulting from optimum choice of ratio of idler to signal frequency.

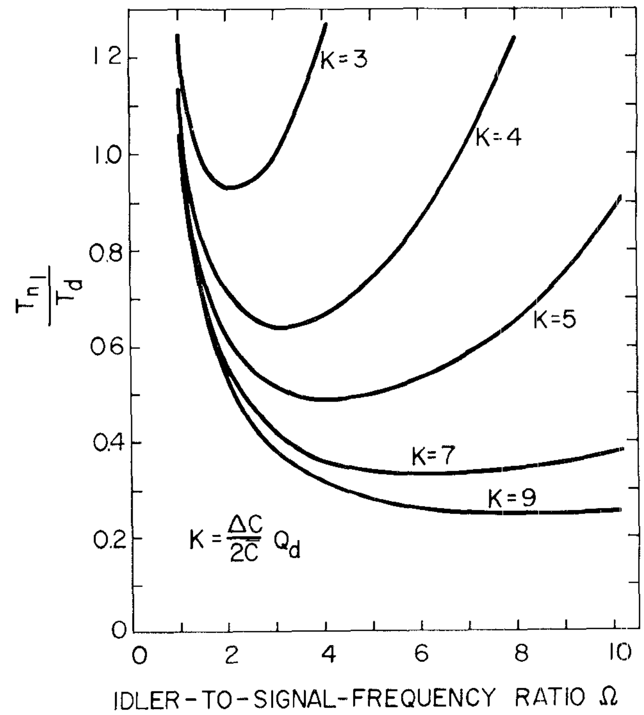

Fig. 3—The effect of the diode parameter K on the variation of the noise temperature (5) with idler-to-signal-frequency ratio Ω.

and the maximum μ_{s_2} that can be achieved with a given diode is

$$\mu_{s_2} \leq K - 1. \tag{9}$$

The minimum realizable radiometer noise temperature is shown in Fig. 2.

Finally, it is of interest to compute approximately the required pump power needed to obtain the results of Figs. 1 and 2. Expressions for the critical pump power P_{cr_1} have been derived elsewhere[3] and are stated here under the assumptions that circuit losses are again negligible and the pump is matched to the diode:

$$P_{cr_1} \geq \frac{\omega_s \overline{C}}{2Q_d^3}\left(\frac{C}{\mathcal{E}}\right)^2\left(\frac{f_p}{f_s}\right)^2 \Omega(1+\mu_{s_1}). \tag{10}$$

This expression can be reduced to normalized form by noting that

$$\frac{\overline{C}}{\mathcal{E}} \simeq \left(\frac{\overline{C}}{\Delta C}\right)\Delta V, \quad \text{where} \quad C = \overline{C} + \mathcal{E}V, \tag{11}$$

where ΔV is the maximum permissible peak-to-peak voltage swing at the pump frequency. Then (10) can be rewritten as

$$\frac{P_{cr_1}}{(\omega_s\overline{C})\left(\frac{\Delta C}{2\overline{C}}\right)(\Delta V)^2} \geq \frac{1}{8}\left(K + \frac{1}{K}\right), \tag{12}$$

where (7), (3), and (4) have been substituted. It is perhaps more instructive to observe how this pump power varies with the optimum frequency ratio. Fig. 4 shows this variation, where both K and Ω are indicated on the abscissa. Similarly, the normalized pump power of the degenerate case ($\Omega \simeq 1$) can be computed. If, instead of μ_{s_1}, μ_{s_2} from (9) is substituted in (10), the normalized pump power for the degenerate case is given by

$$\frac{P_{cr_2}}{(\omega_s\overline{C})\left(\frac{\Delta C}{2\overline{C}}\right)(\Delta V)^2} \geq \frac{1}{2K}. \tag{12a}$$

This is shown by the dashed curve in Fig. 4.

A. Discussion

An examination of these four figures reveals the sensitivity of the amplifier performance to the diode parameter. If one refers, in particular, to Fig. 1 and Fig. 2, the strong dependence on the optimum idler-to-signal-frequency ratio and the resulting minimum noise temperature is evident. It is useful to consider a concrete example. (See Figs. 1, 2, and 4.) For currently available variable-capacitance diodes, a representative number for $\Delta C/\overline{C}$ at some back bias is of the order of 0.6. In order to obtain a relative radar noise temperature of 0.5 (150°K without cooling of the diode), Fig. 2 implies a diode parameter $K=3.7$, from which a diode Q of greater than 12 is calculated. Fig. 1 requires the idler-to-signal-frequency ratio to be about 2.7. The relative radiometer noise temperature for this diode is seen from Fig. 2 to be 0.37. Referring now to Fig. 4, one can make an estimate of the pump power necessary to attain these results. To obtain the radar noise temperature, a relative pump power of 0.48 is needed. Assuming a diode impedance level of 10 ohms and a peak-to-peak voltage swing of 4 volts, approximately 200 milliwatts are necessary (ideally). The corresponding degenerate amplifier needs only about 60 milliwatts. In practice, somewhat larger power is required, either because of losses that have not been accounted for or because of improper matching. In order that (3) or (9) holds, it should be kept in mind that the relations set forth here are valid only if the amplifier has been optimized—optimum coupling to the diode and maximum capacitance swing.

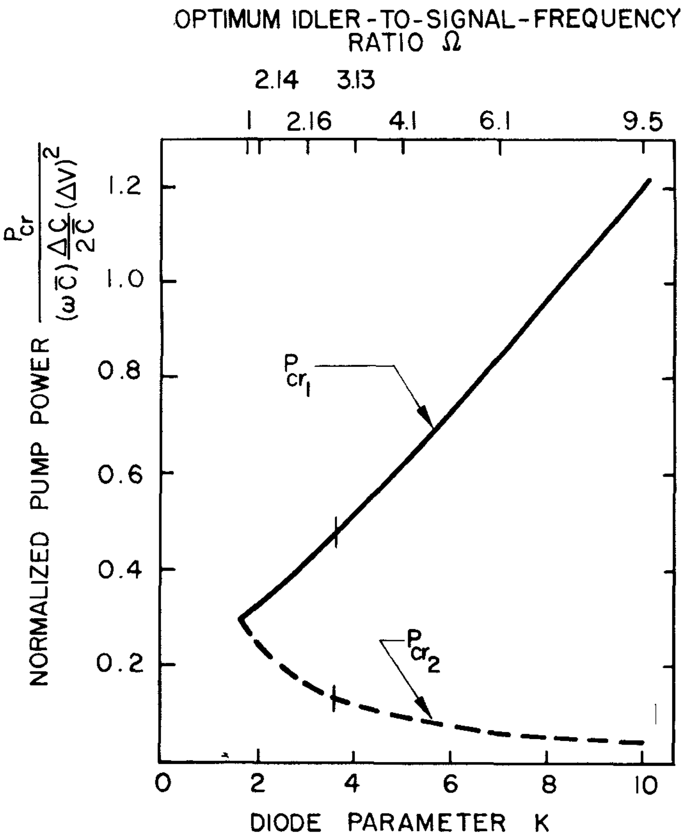

Fig. 4—Normalized pump power required to maintain high gain and the noise temperatures of Fig. 2.

How does this optimum frequency ratio change if, in addition to the diode losses, an external idler termination is admitted? When it is recalled that the diode has been optimized with respect to coupling and capacitance swing to minimize the noise contribution due to its conductance, and the idler-to-signal-frequency ratio has been adjusted to produce approximately equal contributions from both terms on the right-hand side of (5), it is evident that any additional conductance loading, whatever its origin, must of necessity result in a degradation of the noise performance. This, in turn, results in a new idler-to-signal-frequency ratio, which is sufficiently low to equalize again the contribution from the two competing terms in (5). The resulting radar noise temperature T_{n_1} will be higher than that obtained in the absence of the external idler load.

A scheme of this kind can be used to advantage in certain cases. In the above cited example, increase in bandwidth has been achieved at the expense of cooling an external load. If, for example, the diode cannot be cooled, as has been the case with silicon diodes,[5] the benefits resulting from deliberately reducing the idler-to-signal-frequency ratio in order to permit a "heavy" external idler load, which *can* be cooled, may be significant.

It may not always be practical to place the idler frequency at the optimum value, either because of bandwidth considerations (it is usually more difficult to attain bandwidth at higher idler-to-signal-frequency ratios) or because of possible spurious responses. The use of an external idler load in these cases may lead to more flexibility in design and, perhaps, improved performance.

III. Reverse Current-Induced Noise

In order that a parametric amplifier may give the optimum performance, which can be estimated from the knowledge of the figure of merit of the diode [*e.g.*, see (4)], the capacitance of the diode must swing over its entire permissible range. To achieve this maximum swing, the pump voltage (power) is increased until the diode is driven into the forward conduction region, where minority carriers are injected, or into the reverse breakdown region, where either avalanche multiplication or internal field emission limits the maximum voltage and capacitance excursion.[6] The measured noise power, due to reverse current in some diodes, is sufficiently high, even for currents of the order of 5 microamperes, to cause significant deterioration in the noise performance of an amplifier adjusted for optimum performance. Thus, in attempting to optimize an amplifier as outlined in the previous section, one encounters a new limitation to the noise and the bandwidth performance, these quantities being indirectly related when optimization is attempted simultaneously.

A. The Magnitude of Reverse Current-Induced Noise

Reverse breakdown in *p-n* junctions can occur in a variety of ways. Two mechanisms discussed in the literature are the field or tunneling effect[7] (Zener breakdown) and the avalanche or multiplication effect,[8] the latter analogous to the Townsend discharge in gases at high field strength. These two effects can appear as either surface or volume effects. Although either effect may govern the reverse behavior of the junction as dictated by its parameters, the nature of the mechanism is immaterial for the purpose of this discussion, since the primary concern is the magnitude and spectral density of the breakdown-induced noise power relative to thermal noise emanating from the diode. The measured relative magnitude of this noise in a small frequency band in the vicinity of 8.5 kMc is reported here, in an effort to obtain an exploratory estimate in an area where published data are conspicuously nonexistent.

The measurement was performed in the following way. (Fig. 5 illustrates experimental arrangement.) A low-noise parametric amplifier tuned to approximately 8.5 kMc with a bandwidth of 50 Mc followed by a mixer and a 30-Mc IF amplifier, fed a linear power detector, which, in this case, consisted of an automatic noise figure meter operated in the "manual" position. The input to the parametric amplifier was connected alternately to a broad-band matched waveguide termination and to a waveguide diode holder containing a diode adjusted to a particular bias voltage, and matched to the line over a narrow band using a standard *E-H* tuner. The waveguide match was a broad-band termination from which kt watts per unit bandwidth noise power emanated [T = nominal room temperature (300°K)]. When connected to the amplifier input, it provided a power-output reference level, which is to be compared with the alternate input—the matched diode holder. The bias voltage was set to cause various amounts of current to flow through the diode. A voltage-current characteristic for a typical diode is shown in Fig. 6. At each bias voltage, the diode was matched to the line by using both the variable short in the holder and an external *E-H* tuner to achieve a VSWR of 1.02 over a bandwidth in excess of the IF bandwidth. The noise-power-output indication, when the diode holder was switched to the input of the receiver, was compared with the corresponding indication when the matched termination furnished the input. Because of the difference in "noise" bandwidth between the two inputs, it was necessary to established a reference on the basis of which an equiv-

[5] To the best of the author's knowledge, on cooling silicon variable-capacitance diodes the series resistance increases as the temperature is decreased, so as to enhance the thermal noise power emanating from the diode.

[6] These and other noise sources have been suggested as possible causes for the observed increase of measured noise performance above theoretical predictions by M. Uenohara, "Noise consideration of the variable-capacitance parametric amplifier," Proc. IRE, vol. 48, pp. 169–179; February, 1960.

[7] K. B. McAfee, E. J. Ryder, W. Shockley, and M. Sparks, "Observations of Zener current in germanium *p-n* junctions," *Phys. Rev.*, vol. 83, p. 650; August, 1951.

[8] K. G. McKay, "Avalanche in silicon," *Phys. Rev.*, vol. 94, pp. 877–884; May, 1954.

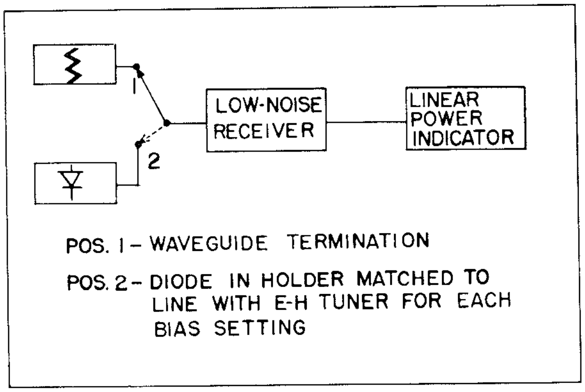

Fig. 5—Experimental arrangement for measuring diode excess noise.

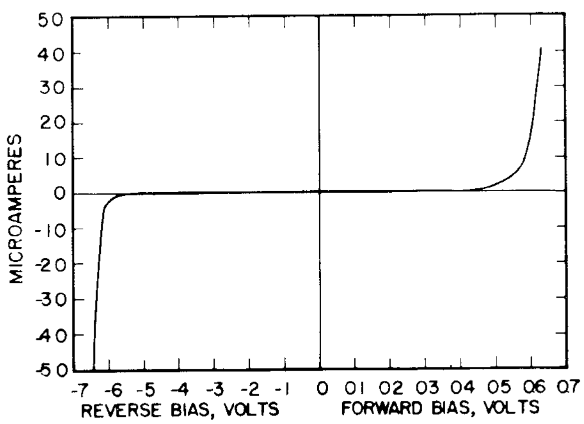

Fig. 6—Typical voltage-current curve of diode used in noise experiment.

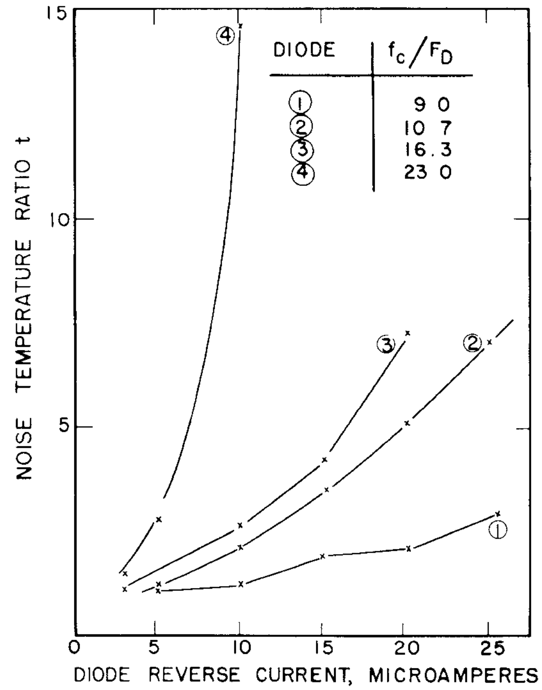

Fig. 7—Noise-temperature ratio t vs indicated diode current. Parameter is f_c/F_D; f_c is cutoff frequency of particular diode, and F_D is noise figure of a parametric amplifier using this diode.

alent "noise-temperature ratio" could be defined. The meaning of this term is as follows: Since, for purposes of evaluating noise performance a matched termination can be described in terms of a temperature[9] (which for a passive device corresponds to the ambient temperature), the noise temperature of a matched active device can be similarly defined. The active device, in this case the diode, then will radiate t times the noise power per unit bandwidth of the passive device. For the purpose of these measurements, the ratio of noise powers resulting from the two inputs when the diode bias voltage was set to zero (no current) was arbitrarily defined as a noise-temperature ratio of unity. Experimentally, this ratio remained constant, as was expected, as long as no current flowed through the diode. The noise-temperature ratio t can be related to the observed ratio of indicated noise powers N and the noise figure of the receiver F as follows:

$$t \simeq (N - 1)F + 1. \tag{13}$$

The measured noise figure for this experiment was 5.6 db. Variation of t with indicated diode reverse current for four silicon mesa diodes is shown in Fig. 7.

In addition to these measurements, an attempt was made to record the variation of the noise-temperature ratio with forward current. This attempt failed in that for currents up to 50 microamperes *no* change was observed in the indicated noise-power output. One may conclude from this that, compared with reverse current on a per unit current basis, the forward current gives rise to a negligible amount of noise, and for this regime a noise-temperature ratio of unity is a good approximation.

B. Discussion

To form some basis for comparison of the "noisiness" of this reverse current, one could naively assume this current to produce shot noise flowing through the series resistance of the diode. In this case, the noise-temperature ratio[10] is given by

$$t \simeq 1 + 20IR, \tag{13a}$$

where I is the current in amperes, which flows through R, the series resistance in ohms. For currents up to the order of milliamperes, the second term in (13a) is negligible. Since the measured contribution of the foward current reported above was indeed negligible, one may conclude that this current produces shot noise.

On the other hand, the situation is vastly different when the current is produced by reverse breakdown.

[9] H. C. Torey and C. A. Whitmer, "Crystal Rectifiers," Rad. Lab. Series, McGraw-Hill Book Co., Inc., New York, N. Y., vol. 15, p. 30; 1948.

[10] In mixer-diode noise theory and practice, this term is often erroneously called noise temperature. Eq. (13a) is valid for a current source causing mean squared shot-noise current $\overline{i^2} = 2eIB$ to flow through a resistance R connected across it.

In this case, it is seen from a glance at Fig. 7 that an equivalent noise-series resistance several orders of magnitude in excess of the actual diode-series resistance must be postulated, in order to account for the large noise temperature resulting from the relatively small current. This probably means that a different mechanism, which gives rise to this excessive noise-power density, is at work. Nevertheless, it is not surprising to find the large noise contribution from the reverse current, since it has long been known that analogous high-field effects in gases and dielectrics have similar properties. A detailed study of the effects of temperature and a measurement of the spectral density over a broad band of frequencies coupled with analysis is necessary for further understanding of this noise-generating mechanism.

An additional observation deserves mention. It is noted from Fig. 7 that the diode with the highest cutoff frequency and the lowest noise figure appears to radiate appreciably more noise per unit current than any other. One can speculate, though perhaps prematurely, that this is as it should be. For a given magnitude of generated noise power in the junction, minimum absorption takes place in the losses of the diode with highest Q and, consequently, maximum radiation results.

C. Effect on Amplifier Performance

It has already been noted that the best amplifier performance that can be obtained with a given diode results when the diode capacitance excursion is maximized. The useful maximum excursion is limited to the region in which no appreciable reverse current flows through the diode. To the extent that the dc voltage-current characteristic of the diode is curved in the neighborhood of reverse-breakdown, the maximum permissible capacitance swing is restricted. This limitation alone is not as serious as it seems, since the capacitance variation near breakdown is quite small and the increase in capacitance swing, which would result from a sharp cutoff voltage-current curve, is not significant in most cases. It is found in some variable-capacitance diodes, however, that reverse current results on the application of the pump voltage, even if the dc bias is adjusted much closer to the forward conduction point than to the reverse-breakdown point. This phenomenon has been observed in our laboratory both at S band and at X band with silicon, as well as germanium, diodes. Recently, a plausible mechanism believed to be responsible for this curious behavior has been suggested.[11] It appears that the lifetime of minority carriers which are injected when the instantaneous voltage swings into the forward-conduction region, can be considerably longer than a period of the pump frequency. The free electron injected into the junction by forward conduction thus remains free as the pump voltage reverses and, during the negative half cycle, precipitates avalanche multiplication through impact ionization. In this way, a net reverse current is produced with its accompanying high noise generation, even though the combined dc and ac voltage is insufficient to establish the critical avalanche field.[12,13] Consequently, in these diodes, the maximum capacitance swing is further restricted. In this case, the limitation is severe, since the relative capacitance swing using the forward voltage region could result in significant increase in the diode figure-of-merit K, particularly in semiconductors with large band-gap voltages.

D. Effect of Refrigeration

It has been amply demonstrated in the literature,[14,15] that the thermal noise power originating in the series resistance of a variable-capacitance diode can, in certain cases, be materially reduced by lowering the diode temperature. These experiments and others conducted in this laboratory proved successful, because the equivalent circuit parameters of the diode did not change materially as the temperature was reduced. Based on these experiments, it can be stated that when refrigeration is applied there will always be an accompanying reduction in noise temperature, provided that the impurity levels below the conduction band and above the valence band are sufficiently "shallow" (*i.e.*, when their "ionization energy" is small compared to kt electron volts at, say, room temperature).

In addition to the beneficial application of cooling described above, a further advantage results because the figure of merit improves at a lower temperature; this is partially caused by the increased maximum permissible capacitance swing. The temperature dependence of variable-capacitance diodes stems primarily from the diffusion potential, which influences the capacitance variation as well as the forward current variation with applied voltage.[16] This diffusion potential is an inverse function of the temperature for the metal-semiconductor junction as well as for the p-n junction. Thus, the point of incipient forward conduction increases with decreasing temperature, and the capacitance at a given bias is reduced accordingly. These predictions are borne out by experiment as shown in Figs. 8 and 9, which show the current-voltage and capacitance-voltage characteristics of a gold-bonded ger-

[11] D. K. Breitzer, *et al.*, "Fifth Quarterly Progress Report, Application of Semiconductor Diodes to Low-Noise Amplifiers, Harmonic Generators, and Fast-Acting TR Switches," Airborne Instruments Lab., Huntington, L. I., N. Y., Rept. No. 4589-1-5, sec. E, Contract 30(603)-1854; September, 1959.

[12] This effect should be absent in diodes that show high rectification efficiency at the particular frequency range involved. An example of this type is a recently developed GaAs point-contact diode.

[13] W. M. Sharpless, "High-frequency gallium arsenide point-contact rectifiers," *Bell Sys. Tech. J.*, vol. 38, pp. 259–269; January, 1959.

[14] M. Uenohara and W. M. Sharpless, "An extremely low-noise 6-kmc parametric amplifier using GaAs point-contact diodes," Proc. IRE, vol. 47, p. 2114; December, 1959.

[15] R. C. Knechtli and R. D. Weglein, "Low-noise parametric amplifier," Proc. IRE, vol. 47, p. 584; April, 1959.

[16] E. Spenke, "Electronic Semiconductors," McGraw-Hill Book Co., Inc., New York, N. Y., ch. 4; 1958.

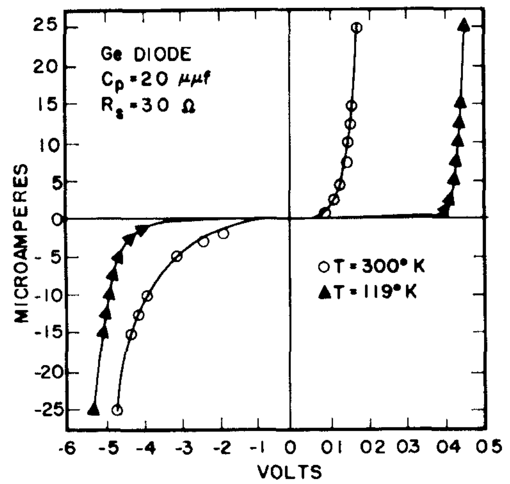

Fig. 8—Temperature dependence of voltage-current curve of germanium alloy junction.

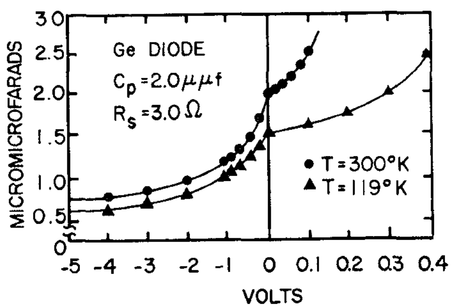

Fig. 9—Temperature dependence of voltage-capacitance curve of germanium alloy junction.

manium diode at two different temperatures. It is seen that the maximum voltage swing at the low temperature is significantly larger than that at room temperature. The resultant increase in capacitance swing, at least from these capacitance measurements (performed at 100 kc), would then be primarily due to the improvement in the reverse-biased region. There is some question as to whether these low-frequency capacitance measurements are valid at microwave frequencies. For example, refrigeration of such a diode in an S-band parametric amplifier experiment[3] did not result in any measurable detuning effects such as would be predicted from the change in capacitance in Fig. 9. It is estimated from these considerations that in some cases more than a 20 per cent increase in the normalized capacitance swing can be realized. In view of expressions (5a) and (8), this increase would lead to an equivalent decrease in the noise temperature.

IV. Conclusion

The two limitations that are discussed in this report will be encountered, if an attempt is made to realize the optimum performance from a given diode. In most practical cases, approach to this optimum diode performance may not be preferred, because of difficulties that arise from increased interdependence of the relevant parameters, such as noise temperature and bandwidth. The previous material should, therefore, serve more as a guide in the design of a particular system, rather than be taken literally, since no attempt has been made to exhaust each subject. Finally, the reader should reflect, once again, on the assumed lumped low-frequency model,[3] which lends validity to the work. This should help to abate disappointment in the discovery that this information may only hold qualitatively at microwave frequencies, where the reader may wish to apply these results.

Glossary of Symbols

G_{ds} = diode shunt conductance at signal frequency.
G_{gs} = effective source conductance at signal frequency.
$\mu_s = G_{gs}/G_{ds}$.
$\omega_i = 2\pi f_i$.
s, i, p = subscripts designating signal, idler, pump frequency, respectively.
$\Omega = f_i/f_s$.
f_c = diode cutoff frequency at given bias.
$Q_d = f_c/f_s$.
$\overline{C}$ = diode mean capacitance.
ΔC = maximum capacitance swing.
T_d = diode temperature (°K).
T_{n_1} = radar noise temperature (°K).
T_{n_2} = radiometer noise temperature (°K).